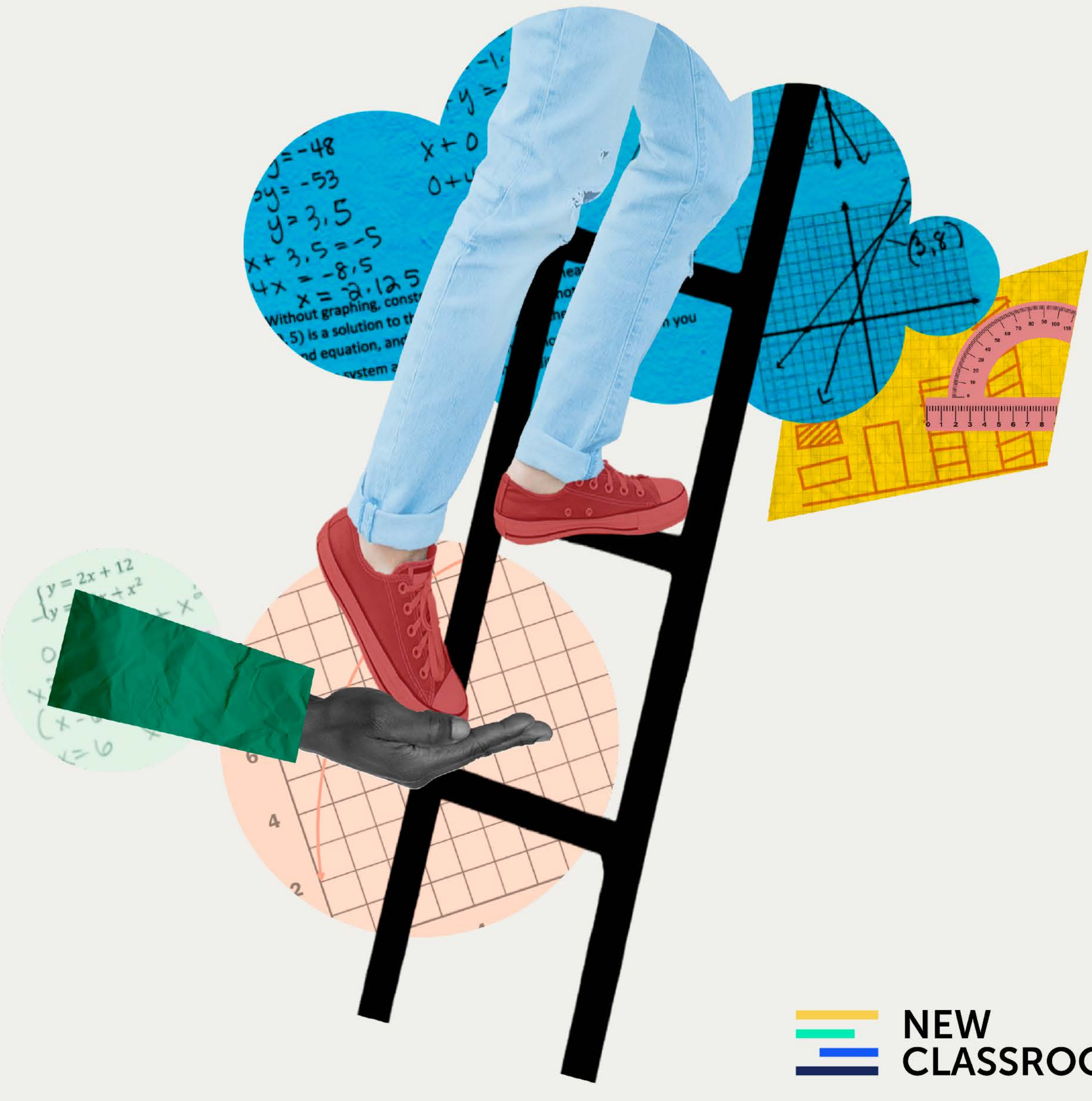
TNTP

Unlocking Algebra

What the Data Tells Us About Helping Students Catch Up



NEW CLASSROOMS

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Introduction

Mathematics learning is a key part of a strong academic foundation that sets students up to thrive in PK–12 schooling and beyond. In *Paths of Opportunity*, TNTP explored the contributing factors of economic and social mobility that enable young people to thrive in school, career, and life. Young people who had access to high-quality academic experiences and excelled in school were more likely to earn a living wage and report high levels of well-being in adulthood.[1]

Algebra I, specifically, is an inflection point in young people's education.[2] The course, generally taken by ninth grade, is a gateway to the higher math courses that set students up for college and careers of their choice, including those in the growing STEM sector.[3] Students who pass Algebra I by ninth grade are more likely to graduate high school, attend college, and earn higher salaries when they enter the workforce.[4]

Unfortunately, access to high-quality, coherent academic experiences in mathematics is far from a given. Too many young people come to Algebra I with significant learning gaps due to inconsistencies and interruptions in their mathematics instruction over several years. Scores from the 2024 National Assessment of Education Progress (NAEP) show that only 28 percent of students across the country were proficient in eighth-grade math.[5]

Since nearly all algebra content draws on knowledge from prior grades, most students enter Algebra I with a lot of ground to make up, and there are real consequences for failing the course. Students who do not complete Algebra I are four times more likely to drop out of high school than those who pass.[6] Given the long-term impact of academic success on young people's lives, it's critical to help more students succeed in Algebra I.

So, we asked the question: How can we help more students master Algebra I, especially those who start the course with significant unfinished learning?

Educators are doing their best to both teach Algebra I content and address each student's learning gaps from prior grades. But since the course builds on such a large body of prior knowledge, it's hard to know how to support students at different levels of algebra readiness.

What's more, few districts and schools have a coherent instructional program, where the curriculum, materials, interventions, and assessments all work together to advance the same set of grade-level expectations. Whole-class instruction (Tier 1) and intervention (Tier 2) often run on separate tracks. So how can teachers and intervention specialists work together more effectively to support students in Algebra I?

To find out, we analyzed three years of data from more than 2,000 math students who used an online learning platform by New Classrooms called Teach to One Roadmaps as a supplement to their core Algebra I classes.[7] In Roadmaps, students practiced both grade-level Algebra I content and algebra-related concepts and skills from prior grades, which gave us detailed data on how students learned concepts and skills over time. (Figure 1)

Figure 1 | Teach to One Roadmaps Algebra Ladder

	Algebra Ladder **All concepts and skills** related to Algebra throughout a student's math education, both in Algebra I and prior grades.	**Predecessor** **A specific concept and skill** in the Algebra ladder, that is academically foundational to another concept and skill.
Algebra I	**52**	Average Rate of Change
Grade 8	**28**	Construct Functions
Grade 7	**19**	Proportion Graphs
Grade 6	**21**	
Grade 5	**11**	Coordinate Plane
Grade 4	**11**	
Pre-4	**5**	
	Number of Algebra-related concepts and skills, by grade.	Individual Algebra-related concepts and skills, by grade.

There is no universal list of Algebra I concepts and skills. States describe and structure their math standards in slightly different ways, and standards cover different amounts of material. Researchers and curriculum providers have also constructed slightly different lists of concepts and skills. However, most broadly agree on what students need to learn.

Definitions used in this research:

1. **Skill.** A mathematical competency (including procedural, conceptual, and/or applied understanding) that can be taught in a 30- to 40-minute class session and assessed with a short quiz. A skill from any grade can be taught at the procedural, conceptual, or applied level.
2. **Algebra Ladder.** Approximately 150 skills in Teach to One Roadmaps that are related to algebra throughout a student's math education. Of those, about 50 skills are part of grade-level expectations in Algebra I, and the others build toward algebra in earlier grades.
3. **Predecessor.** A specific skill in the algebra ladder that is academically foundational to another skill. A predecessor skill can come from the same grade or a previous grade.
4. **Key Predecessors.** Up to three predecessors that most strongly predicted student success on a new concept and skill, based on our analysis. Each concept and skill has its own set of key predecessors. See Appendix 2 for more details.

Here's What We Found:

1. **Algebra proficiency improves when students learn new algebra-related concepts and skills, including those from prior grades.** Many students who entered Algebra I knew only one-third of the algebra-related concepts and skills from prior grades. As these students learned more concepts and skills related to algebra—both from their current grade and from previous ones—they performed better on state tests.
2. **Learning new algebra-related concepts and skills requires applying key predecessor concepts and skills.** Students who started Algebra I with unfinished learning didn't need to learn every concept and skill they missed in prior grades before attempting something new. Instead, learning the most critical predecessors meaningfully increased their chances of acquiring new algebra-related concepts and skills.
3. **Tier 2 support is most effective when students build on what they know.** Over the course of a year, students who start with unfinished learning can make up the most ground when their Tier 2 support is tailored to their knowledge of key predecessors. In this approach, each student works on the most advanced algebra-related concepts and skills, at any grade level, that they are best equipped to learn.

This analysis indicates that schools can use Tier 2 support to meaningfully accelerate learning in Algebra I, regardless of what students know coming into the course. Acquiring these key predecessors at the right time—either to support the current Tier 1 lesson or in advance of an upcoming lesson—can help more students catch up.

That said, teachers and students can't do this alone. They need support from leaders at the school, district, and state level. The education system must ensure a high-quality shared curriculum, holistic planning for Tier I and Tier 2 instruction, and mechanisms for consistent communication and collaboration between educators.

This paper offers our starting list of key predecessors to support Algebra I instruction in individual classrooms. It also shares recommendations for educators and leaders on how to help school systems ensure that more young people can access life-changing mathematics learning.

Findings

1. Algebra proficiency improves when students learn new algebra-related concepts and skills, including those from prior grades.

Many students who entered Algebra I knew only one-third of the algebra-related concepts and skills from prior grades. As these students learned more concepts and skills related to algebra—both from their current grade and from previous ones—they performed better on state tests.

What does it take for students to succeed in Algebra I? In an ideal world, students who enroll in an Algebra I course would bring with them all of the foundational algebra-related concepts and skills that were taught in prior school years. But when students start the course with unfinished learning, how many algebra-related concepts and skills do they need to learn to meet the expectations of their grade?

To find out, we matched student progress in Teach to One Roadmaps with end-of-year state-administered tests, using data from multiple grades to expand the sample size. Of students using Roadmaps, 201 Algebra I students and 349 students in grades 5–8 also had state testing data available. To compare student performance across different state tests, we compared each student's raw test score to the score needed to meet their state's test expectations (typically a performance level of at least a 4 out of 5).

As students mastered more concepts and skills overall, both from their current grade and prior ones, they performed steadily better on end-of-year assessments for their respective grades. (Figure 2) This held true regardless of how much students knew at the start of the year.[8]

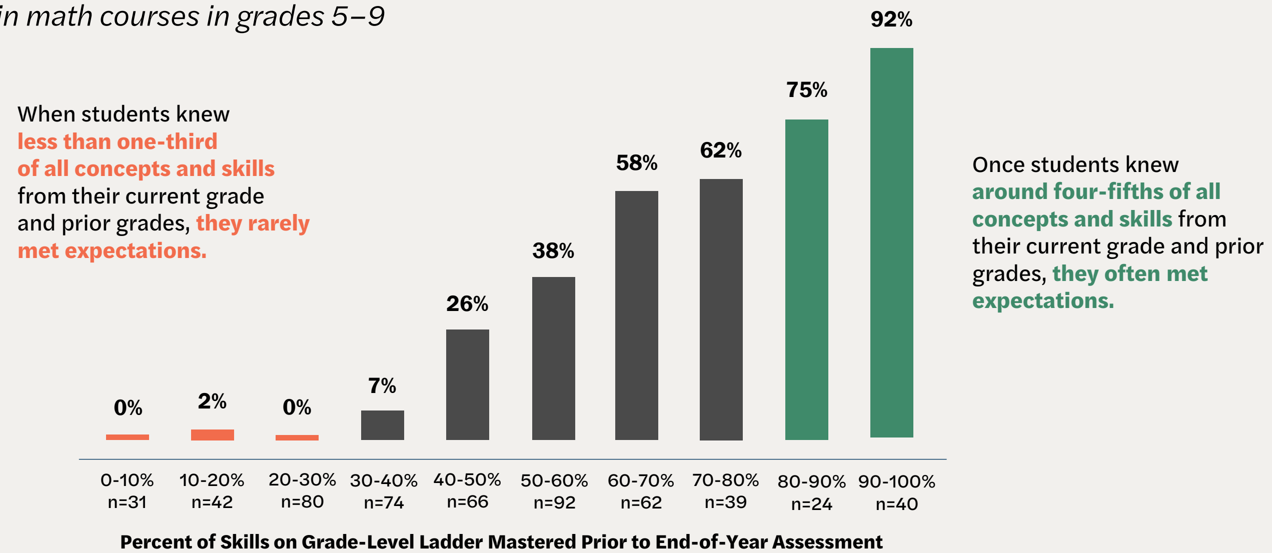
This is important for two reasons. First, it confirms that assessed progress in Roadmaps is a reliable proxy for overall student learning. Second, it helps us understand how much progress young people need to make to meet the expectations of their grade.

Once students knew at least four-fifths of all required concepts and skills—both from their grade and prior grades—they were likely to "meet expectations" on state tests. The converse was also true: When students knew less than one-third of all these concepts and skills, they almost never met expectations.[9]

Here's the core challenge: Almost half of the students in our full sample started Algebra I knowing just one-third of all the algebra-related concepts and skills from prior grades.[10] That's a lot of ground to make up.

Figure 2 | Percent Meeting State Test Expectations, by Proportion of Concepts and Skills on Grade-Level Ladder that were Mastered

Students in math courses in grades 5–9



Note: Meeting expectations is defined differently in each state, but typically requires earning a performance level of at least a Level 4 out of 5. In the plot above, percentages that are exactly equal to the upper bound of the bin (e.g., 10%, 20%, 30%, etc.) are included in the next highest bin. We also looked at the correlation coefficient between student's standardized test score—that is, their test score relative to the state's threshold for meeting expectations, divided by the statewide standard deviation—and the number of total skills mastered, and found it was 0.73 *** ($p < 0.001$).

2. Learning new algebra-related concepts and skills requires applying key predecessor concepts and skills.

Students who started Algebra I with unfinished learning didn't need to learn every concept and skill they missed in prior grades before attempting something new. Instead, learning the most critical predecessors meaningfully increased their chances of acquiring new algebra-related concepts and skills.

If students with unfinished learning need to acquire a critical mass of algebra-related concepts and skills to succeed in Algebra I, what's the most efficient way for them to learn new concepts and skills?

Math concepts and skills stack over time, and a new concept and skill usually builds on predecessors—academically foundational concepts and skills introduced earlier on the algebra ladder. A typical problem in Algebra I has dozens of predecessors, some that date back to elementary school. If a student can't yet access an Algebra I concept or skill, which predecessors are most likely to help?

This can be hard for teachers to answer. While resources like coherence maps show all possible connections between various concepts and skills, the sheer number of options can be overwhelming. Without a good way to prioritize predecessors, teachers are left to guess on their own.

To narrow the options, we looked at which specific predecessors most influenced students' success in Roadmaps. We analyzed more than 125,000 individual student attempts on an algebra-related concept and skill, each captured in a 24-hour period, both from Algebra I and prior grades.

We looked separately at each concept and skill, accounting for where students started the year, the number of previous attempts at that skill, and the total number of predecessor skills the students had already mastered. This gave us consistent snapshots of one-day student success rates on each concept and skill.

We asked: All else being equal, what specific predecessor knowledge made a student most likely to succeed on a new Algebra I concept and skill?[11]

For each Algebra I concept and skill, we identified up to three key predecessors. These predecessors most strongly predicted student success on any given attempt. (Appendix 1) This is not a definitive list, but the trends are broadly applicable: Knowing a small set of key predecessors made students more likely to master a new concept and skill. This held true across all students and all schools.[12]

Key predecessors to grade-level concepts and skills were usually introduced within the last two years. Most (84 percent) of the key predecessors to Algebra I concepts and skills were introduced in seventh grade onward.[13] For example, the Algebra I concept and skill, average rate of change, has dozens of predecessors from prior grades, but all three of the key predecessors that emerged in this analysis were introduced in eighth grade. (Figure 3)

Figure 3 | Predecessors for Average Rate of Change (Algebra I)

Predecessor: All concepts and skills in Teach to One Roadmaps that are foundational for average rate of change.
Key Predecessor*: The concepts and skills that most strongly predicted student success on average rate of change.

Grade 8	• **Construct Functions*** • Compare Proportions • Define Functions • **Function Rules***	• Graphs in Context • Plot Linear Functions • Slope and y-Intercept • **Slope Given 2 Points***
Grade 7	• Add Integers • Add Opposites • Creating Expressions & Equations • Multiply and Divide Integers • Proportion Graphs	• Proportion Equations • Ratios and Proportions • Solve 2-Step Equations with +/- Numbers • Solve 2-Step Equations • Subtract Integers
Grade 6	• 1-Step Equations Word Problems • Coordinate Plane in 4 Quadrants • Equivalent Ratios • Ordering +/- Rational Numbers • Ratios • Represent Linear Relationships • Represent Positive and Negative Numbers	• Solve 1-Step Equations • Substitution • Unit Rates • Variables and Expressions • Understand Absolute Value • Understanding Equations
Grade 5	• Coordinate Plane Problems • Coordinate Plane • Divide Decimals by Whole Numbers • Multiply by Powers of 10 • Multiply and Divide by Powers of 10	• Fractions as Division • Numerical Patterns • Order of Operations Without Exponents • Simplify Fractions
Grade 4 or Below	• 2-Step Word Problems • Addition and Subtraction Equations • Distributive Property • Equivalent Fractions • Line Relationships • Generate Patterns • Identify Factors of Whole Numbers • Multiples of Whole Numbers	• Multiplication and Division Equations • Multiplicative Comparisons • Multiply 2-Digit Whole Numbers • Operation Properties • Rectangular Area • Understanding Area • Whole Number Word Problems

What does this mean for the students who started Algebra I with significant unfinished learning?

Around half of our sample started Algebra I with unfinished learning that dates back multiple years. They were likely to be missing not only the key predecessors from Algebra I, but also the predecessors from prior grades.

It's no surprise that when these students tackled a new Algebra I concept and skill knowing few predecessors—and no key predecessors—they only succeeded around 1 in 10 times on any given attempt (13 percent). (Figure 4)

Learning more predecessors overall created some learning boost. Students who had mastered most of the predecessors but fewer than half of key predecessors were successful 3 in 10 times (31 percent) on any given attempt. But learning all key predecessors nearly doubled students' chances of success. When students knew all the key predecessors, they were successful nearly 6 in 10 times (58 percent) on any given attempt.[14]

In other words, students who started Algebra I with unfinished learning didn't need to learn every concept and skill they missed in prior grades before attempting something new. Instead, learning the most critical predecessors meaningfully increased their chances of acquiring a new algebra-related concept and skill.[15] The more key predecessors they knew, the faster they could acquire new concepts and skills, helping them catch up.

Figure 4 | Success Rate on Any Single Attempt at Algebra I Concepts and Skills

Students with unfinished learning from prior grades

Few predecessors, no key predecessors

When students knew fewer than half of the predecessors and no key predecessors, their success rate on a new Algebra I concept and skill was:

13%

Most predecessors, few key predecessors

When students knew at least 80% of the predecessors, but fewer than half of the key predecessors, their success rate was:

31%

Most predecessors, all key predecessors

When students knew at least 80% of the predecessors and all the key predecessors, their success rate was:

58%

Note: Students with unfinished learning are those who began the year with 75 or fewer concepts and skills. There were 1,303 attempts made when students had few predecessors and no key predecessors. There were 1,627 attempts made when students had at least 80% of the predecessors but less than half of the key predecessors. There were 5,315 attempts made when students had mastered all key predecessors. Note that attempts where all key predecessors were mastered also tend to have more predecessors mastered overall (average = 98% predecessor mastery rate), but these results were similar when we looked only at attempts where all key predecessors were mastered but 90% or fewer of all predecessors were mastered: The success rate was 66% and N = 254

3. Tier 2 support is most effective when students build on what they know.

Over a year, students who start behind can make up the most ground when their Tier 2 support is tailored to their knowledge of key predecessors. In this approach, each student works on the most advanced algebra-related concepts and skills, at any grade level, that they are best equipped to learn.

Learning key predecessors is critical, but each key predecessor requires its own set of predecessor knowledge. For students with significant unfinished learning, focusing only on the key predecessors for Algebra I concepts and skills may not be sufficient. These students often need extra support in Tier 2 instruction, which may be provided by core classroom teachers, intervention specialists, tutors, or digital learning platforms.

So how can teachers and specialists best support students at different levels of algebra readiness? What overall instructional approach should they take for Tier 2 support?

We looked at only Algebra I students with unfinished learning and simulated their success in Roadmaps with three different instructional strategies over a year[16]:

1. **A start-at-the-bottom approach** broadly backfills all the concepts and skills students are missing from prior grades. If a diagnostic assessment shows that a student is missing algebra-related concepts and skills from sixth grade, they are said to be "learning at a sixth-grade level" and that student reviews sixth-grade content.
2. **A grade-level-only approach** sticks purely to Algebra I concepts and skills in Tier 2 intervention. This assumes that students benefit from more exposure to grade-level content and that they can pick up the fundamentals as they go.
3. **An individualized approach** tailors the Tier 2 support to each student. Learning is focused on key predecessor gaps that students are academically ready to learn, and that relate to what's being taught during Tier 1 instruction.

To build the simulation, we analyzed more than 45,000 individual attempts by nearly 1,800 Algebra I students on all concepts and skills in the full algebra ladder, both from Algebra I and prior grades. We know which concepts and skills these students had mastered at the start of the year. If these same students experienced different approaches to Tier 2 instruction, how different would their learning be by the end of the year?

We used each student's actual track record (accounting for their starting knowledge and mastery of key predecessors) to predict how likely they would be to succeed on a new concept and skill.[17] Then we simulated what would happen if each student made 50 attempts on a new algebra-related content during a school year (tackling one or two new concepts and skills per week in the school year), using the three different approaches: start at the bottom, grade level only, and individualized. (Figure 5)

Figure 5 | Simulated Student Success with Different Tier 2 Approaches

We used data from real Algebra I students with unfinished learning and simulated their success with different instructional approaches over a school year. This assumes 50 attempts on algebra-related concepts and skills, either from Algebra I or prior grades. From those attempts:

Approach	Success Rate on Attempts	Total Concepts & Skills Gained
Start at the Bottom **Students work only at the grade level where they're missing most prior concepts and skills.** They work straight through all concepts and skills in that grade and repeat some content they've already covered.	45%	12
Grade Level Only **Students work only on concepts and skills introduced in Algebra I.** They work straight through the Algebra I curriculum. Whether or not they master a concept or skill, they move on to the next. Nearly all content is new to them.	28%	14
Individualized **Students work on the most advanced concept and skill that they're ready to learn.** First, the simulation identifies the concepts and skills where students had mastered the most key predecessors, then it assigns problems from the highest possible grade level. All content is new.	50%	25

Note: Predictions based on estimates from a linear probability model predicting attempt success controlling for the proportion of key predecessors mastered, the number of skills mastered at the beginning of the year (BOY) either through a diagnostic assessment and/or previous years in the system, and their interaction. All students in an Algebra I course were included in model, but simulations based only on those who began the year with 75 or fewer skills. For each attempt at a skill, we calculated the probability of success using the estimated coefficients and the student's personal BOY skills and the proportion of key predecessors that student had already mastered. We then used these probabilities in a random binomial draw. Each student's collection of mastered skills was updated after each of the 50 iterations of the simulation so that their key predecessor rate on future skills could change based on the results of the simulation. To simulate the "Start at the Bottom" approach we took the mean grade-level of all BOY skills and rounded to the nearest integer grade to get the student's initial grade-level. Across the 50 attempts, students began with the first skill in their assigned grade-level and moved to the next one regardless of simulated attempt outcome. For the "Grade-Level Skills Only" approach, all students began at the first skill of 9th grade Algebra and moved to the next skill after each of the 50 attempts, regardless of outcome. Skills in all grade-levels were ordered so that students never attempted a skill in a grade without first attempting its same-grade predecessors. For the final approach, each skill was sorted by the proportion of key predecessors already completed and then arranged by descending grade-level. When there were ties, we randomly selected a skill for a student to attempt. Students could attempt the same skill if they did not pass it on a simulated attempt. Though mastering some skills allow one to infer mastery of other, closely related predecessor skills, we did not bestow inferential mastery over any skill given this was a simulation.

In our simulation, the two blanket approaches—starting at the bottom and providing only grade-level content, regardless of readiness—both yielded limited results. Neither of these approaches took into account what students already knew.

In the **start-at-the-bottom approach**, students gained only 12 new concepts and skills after 50 attempts. The concepts and skills were often repetitive but were generally accessible, with students succeeding in almost half of the attempts. When students worked straight through a previous grade's content, they wasted time on some concepts and skills they already knew.

In the **grade-level-only approach**, students fared a bit better, but not by much. They gained an average of 14 new skills after 50 attempts. The concepts and skills were new but often too challenging, and students were only successful in about 1 in 4 attempts.

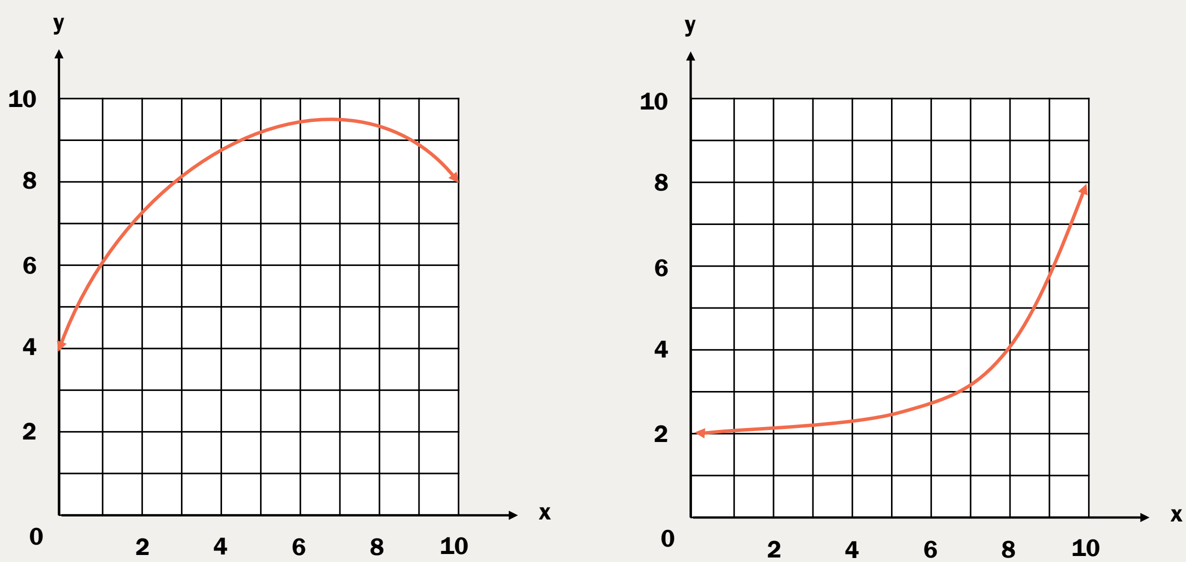
Students learned most at the sweet spot between challenge and readiness. When the **individualized approach** responded to students' existing mastery of key predecessors, they gained an average of 25 new concepts and skills for every 50 attempts. The content was still challenging; even when building on key predecessors, they were successful only half the time. But compared to the other two approaches, students learned nearly twice as much over the course of a year.

This is obviously a simplified scenario, and most educators don't have real-time recommendations on key predecessors. Yet many do have student diagnostic data in some form, including assessment data from prior grades.[18] As a starting point, teachers and specialists can use existing learning data to provide more individualized Tier 2 support. Also, since key predecessors for Algebra I concepts and skills tend to come from seventh grade onwards, teachers can draw first from these grades when students need a refresher. The goal is to provide support by concept and skill rather than by grade level.

Finally, this simple Tier 2 scenario doesn't try to measure the total number of concepts and skills that students could learn over the course of a year in both Tier 2 and core instruction. It is meant to illustrate the relative efficacy of different instructional approaches and can't capture everything required to help students catch up. What it can do is help focus support for students on the concepts and skills that give them the biggest learning boost. Figure 6 shows what that might look like in a real classroom.

Figure 6 | Three Approaches in Action

In Algebra I, students learn how to determine the ***Average Rate of Change*** for linear and non-linear functions, expressed in equations, tables, and graphs. *For example, which function has the greatest average rate of change over the interval 0 ≤ x ≤ 10?*



Two students who have yet to learn this concept:

Ana can make conceptual sense of the graphs but gets confused by representing linear functions in equations (a key predecessor from grade 8). In a grade-level-only approach, Ana would just keep trying the grade-level problem—and she might get it, or she might get more confused and discouraged.

Ben can do the process steps to solve equations but doesn't fully understand the concept of rate of change and how to interpret it on a graph (non–key predecessors from grades 5 and 7). In a start-at-the-bottom approach, he might practice graphing problems from grade 5, which are too simple to be helpful.

In an individualized approach, Ana and Ben each focus on the predecessors for which they are uniquely ready.

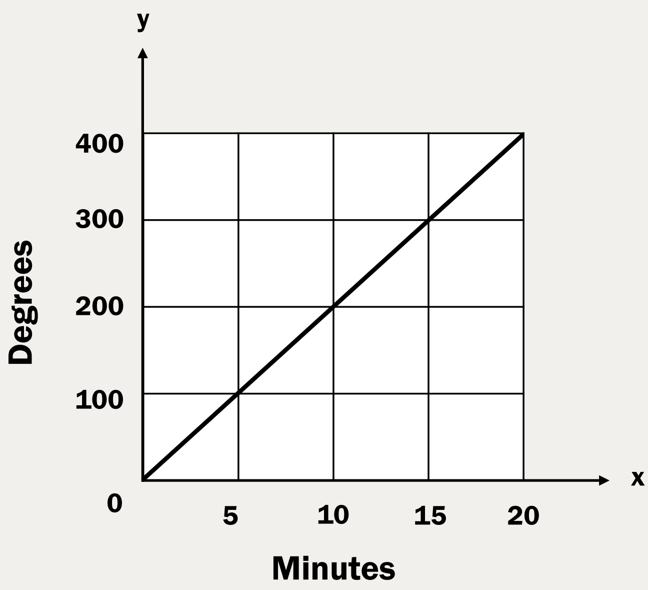
Ana works first with the **grade 8 key predecessor** on constructing linear functions in different ways (in an equation, a table, and a graph). The problem below could help her understand the equations for a steady rate of change, which sets her up to learn nonlinear equations.

To ship a package, a company charges a one-time fee plus a fee based on the weight of the package. This table shows the total shipping costs for four packages of different weights. What equation represents the total shipping cost, C, in dollars, of a package weighing W pounds?

Shipping Costs	
Weight of Package (pounds)	Total Shipping Cost (dollars)
4	$11.00
8	$17.00
12	$23.00
16	$29.00

Ben works first with the **grade 7 predecessor** on proportional relationships. The problem below could help him understand the simplest version of rate of change. Once he better understands slope and rate of change in a linear equation, he may be ready to jump directly back into the work of the Algebra I standard.

John drew this graph to represent a frozen pizza cooking in an oven. What statement could describe the situation John graphed?
(A) The pizza temperature increases 5 degrees every minute.
(B) The pizza temperature increases 20 degrees every minute.



These are just two examples of support for this concept and skill, and other examples might draw on different predecessors from different grades. The goal is for students to work on the most challenging concept and skill that they are ready to learn.

Source for test item: https://cortex.ttohome.org/skills/primer/554

Recommendations

Helping students succeed in Algebra I starts with access to grade-appropriate assignments and high expectations in core classes. *The Opportunity Myth* showed that far too many students were not ever given the opportunity to engage with work appropriate for their grade. This had a devastating effect on student learning, particularly for students who started the school year behind.[19]

Yet *The Opportunity Myth* did not argue that students should only ever access grade-level material. High expectations are not enough; students also need a pathway to meet those high expectations. Whole-class instruction (Tier 1) should set a high standard for all students; intervention (Tier 2) should respond to each student's readiness and equip students to engage with the current lesson in a larger group setting. As this analysis shows, when students work at the sweet spot between challenge and readiness, they learn the most algebra over time.

To hit that sweet spot, teachers and Tier 2 intervention specialists need timely diagnostic data and an easy way to identify key predecessors. In Appendix 1, we share the list of key predecessors for Algebra I that emerged from this research. This list is not meant to be definitive, but we hope it is helpful as a starting place. We encourage the field, including fellow researchers and providers of digital learning platforms, to use their own student data to expand on this research and refine this approach.

Teachers and specialists also need to work within a system of instructional coherence, described in *The Opportunity Makers*. Coherence means that all pieces of the school's instructional program—curriculum, materials, interventions, and assessments—work together to advance the same set of grade-level expectations. Students know exactly what to expect and how to succeed.

Unfortunately, this level of instructional coherence is rare. In many schools, Tier 2 materials only loosely connect to core classes, and assessments measure different things in each tier. Core teachers and specialists may not have time to meet or connect their lessons. As a result, the students who receive the most support typically have the most disjointed experiences at school.[20]

Instructional incoherence isn't intentional. Often, it's the result of well-intentioned people trying to do too many things or working within real constraints. But the costs of incoherence are real. Success in school, and in Algebra I specifically, has a long-lasting impact on the lives of young people. Below, we've outlined steps that educators and leaders can take to begin building more coherent academic experiences that set Algebra I teachers and students up for success.

Recommendations for Schools:

Ensure Tier 1 and Tier 2 Work Together

Our analysis shows that students with significant unfinished learning can begin Algebra I with, on average, no more than 32 of the 95 algebra-related concepts and skills from prior grades. These students would need to acquire around 60 new algebra-related concepts and skills in Tier 2 instruction, in addition to what they learn in core instruction, to materially increase their chances of achieving proficiency on the end-of-year Algebra I assessment.

Making up that much ground is not easy. To meet this ambitious target, there must be a meaningful investment in coherent Tier 2 support. At the school level, leaders must set clear expectations that the purpose of Tier 2 is preparing students for core instruction. That may mean adjusting the schedule so that core teachers and Tier 2 intervention specialists have time to work together; more time is set aside for individualized support in core math instruction; and double blocks (80-90 minutes) for Algebra I can be created.

At the student level, there must be individualized support and an intentional focus on key predecessors. Students with unfinished learning would need to master at least two new concepts and skills a week in Tier 2, for over 30 weeks, to acquire around 60 new algebra-related concepts and skills. Concepts and skills would vary by student, based on each one's unique understanding of key predecessor skills, and should relate to what's being taught in Tier 1.

This ambitious Tier 2 target is a worthwhile benchmark to help more current Algebra I students pass the course by ninth grade, particularly those who start furthest behind. However, our analysis also clearly illustrates the importance of students having access to coherent, high-impact math instruction at each grade level to ensure they learn foundational concepts and skills along the way. Teachers at multiple grade-levels have a role to play in preparing students for Algebra I.

Key questions for school leaders and educators to consider:

1. Is Tier 2 instruction differentiated to support each student in acquiring new concepts and skills in ways that account for their key predecessor knowledge?
2. How does Tier 1 instruction inform instruction in Tier 2, and vice versa?
3. Do we have adequate time built in for Tier 2 instruction? If that requires different teachers, is there time for them to collaborate?
4. Do students and families clearly understand the skills necessary to achieve proficiency? Do they have access to the tools and supports required to bolster learning outside the classroom?

Recommendations for School Systems:

Adopt Rigorous Tier 2 Solutions and Provide Supports for Teachers

School systems often use digital learning products as one part of their Tier 2 strategy. But many popular platforms take a start-at-the-bottom approach, providing practice that is too easy or unrelated to the lesson at hand. Some take a grade-level-only approach that aligns to Tier 1 instruction but does little to address unfinished learning on key predecessor concepts and skills. Others may focus on a common set of predecessors for all students, rather than taking an individualized approach that meets each student where they are and provides them with a path to proficiency. When purchasing a Tier 2 solution, it's important for system leaders to understand how the platform diagnoses student readiness and recommends practice tasks. In addition, school systems should consider future research that examines how we can sustain these recommendations in practice.

Key questions for school system leaders to consider:

1. How does the Tier 2 platform integrate with Tier 1 instruction?
2. Does the Tier 2 platform provide teachers with precise diagnostics to identify the most relevant unfinished learning for each student? Can teachers easily assign modules that address their students' unique sets of key predecessor gaps and that relate to Tier 1 instruction?
3. How is high-quality content on predecessor skills used, including content from prior years? How far back can students go in order to address key predecessors?
4. Is there a clear and transparent pathway to proficiency for each student, informed by key predecessors, and real-time progress on that individualized pathway?
5. What supports and/or resources must be in place for Algebra teachers to ensure their students receive Tier 2 instruction when needed? How can there be improved coordination between teachers and their roles in supporting students?

Recommendations for States:

Create the Conditions for Coherent Tier 1 and Tier 2 Experiences

Currently, there's a lot of focus on rigorous Tier 1 curriculum, and many states have set adoption criteria for high-quality instructional materials. However, few states set clear parameters for Tier 2 experiences. State leaders can use Tier 1 curriculum lists to advocate for coherent interventions (e.g., Tier 1 materials must be adopted with appropriate supports) or add a dedicated Tier 2 list (e.g., solutions that are aligned to Tier 1 curricula).

Alignment doesn't necessarily mean that Tier 1 and 2 materials are from the same provider, or that students work only on grade-level content in both settings. Instead, Tier 2 should address unfinished learning and build directly toward grade-level content

Key questions for state education leaders to consider:

1. Have we signaled to our local education agencies that Tier 2 instruction should supplement Tier 1 and address unfinished learning to prepare students for grade-level work?
2. Have we provided clear guidance for Tier 2 materials, including tools that:
   - Integrate with Tier 1
   - Precisely diagnose and address unfinished learning
   - Include content from prior grades
   - Provide an individual pathway to proficiency for each student based on their predecessor knowledge.

Appendix

Appendix 1: List of Key Predecessors

This analysis identified key predecessors for the 52 Algebra I concepts and skills in Roadmaps. We used the Roadmaps student dataset to identify up to three predecessors for each concept and skill that most strongly predicted student success on that concept and skill. This analysis surfaced 151 key predecessors.[21]

However, this analysis also has some limitations. It is based on pure statistical probability from a single student dataset. Separately, New Classrooms also ranks predecessors in order of importance, drawing on existing thinking in the field about which concepts and skills are academically foundational to others.

For this reason, we cross-referenced our list of key predecessors generated by the analysis with the New Classrooms list. The 101 key predecessors, below, rise to the top as most important in both lists.[22]

These key predecessors are not meant to be definitive, and we encourage others to build on and refine this approach. Instead, it's a jumping-off point. *This list* is meant to help educators provide more targeted support to students learning specific Algebra I concepts and skills.

More details on each individual concept and skill can be found on the website for *Teach to One Roadmaps*.

Appendix 2: Research Methodology

Data and Definitions

Our analysis used data from students who engaged with Algebra I content in New Classrooms' online learning platform, Teach to One Roadmaps, which supplements core instruction. Students begin with a short diagnostic assessment to determine which algebra-related concepts and skills, both from Algebra I and prior grades, they have already mastered. It then suggests a personalized learning roadmap of algebra-related concepts and skills to then focus on during the school year that build towards grade-level proficiency. Students can follow this roadmap as suggested but can also choose different skills if desired or directed by their teacher.

As students engage with concepts and skills in Roadmaps, they can learn about the concept and skill, watch example videos, and complete practice problems. Critically, they also complete short assessments to check if they've successfully mastered the concept and skill. If they pass the assessment, they advance to the next concept and skill; if not, they're guided to recommended supporting concepts and skills. This comprehensive record of attempted algebra-related concepts and skills provided detailed data on how students build knowledge over time and was the foundation of our analysis.

We examined all attempts to demonstrate mastery over algebra-related skills made by students in an Algebra I classroom in the 2021-22, 2022-23, and 2023-24 school year.[23] This provided a record of 128,579 attempted or mastered skills from 2,207 Algebra I students.[24]

Using this record of skill completion, we developed the following key definitions throughout our analysis:

1. **Diagnosed Concepts and Skills:** Using the results of the beginning-of-year diagnostic assessment, we could identify how many of the 147 concepts and skills in the algebra ladder students had already mastered.[25] Of those, 52 were introduced during Algebra I, and 95 were introduced in prior grades. In some cases, students beginning Algebra I used Teach to One Roadmaps in earlier grades, so we could also use historical data to detect concepts and skills already mastered.

2. **Students with Unfinished Learning:** A student who was fully on track would enter Algebra I with 95 algebra-related skills from prior grades. We considered any student who began Algebra I having mastered no more than 75 skills to have unfinished learning. Most students (87 percent) began Algebra I with unfinished learning. Almost half (46 percent) began Algebra I with no more than 32 concepts and skills, on average.

3. **Assessed Skills and Concepts:** During the school year, each concept and skill is assessed with a five-question test; students pass and are considered to have mastered the skill when they answer 4 of 5 questions correctly.

4. **Attempts:** A student's performance on the assessment on a single school day. For each concept and skill, we counted one attempt per day, taking students' best result, to account for students who use the assessments repeatedly as practice.

Connecting Algebra I Concept and Skill Mastery to State-Tested Achievement

For a subset of students using Teach to One Roadmaps, we also had access to their associated end-of-year or end-of-course state-administered assessment. This allowed us to compare the extent to which the proportion of concepts and skills mastered by the end of the year was associated with an external and high-stakes assessment.

Our interest was in the broad connection between concept and skill completion as assessed by Teach to One Roadmaps and tested achievement, so we included all available mathematics students with state testing information, not just those in Algebra I classrooms. This included students in grades 5–9. For each student, we calculated the proportion of their appropriate grade-level ladder (sixth-grade students were compared to the sixth-grade mathematics ladder) they had completed at the time they took their state test. In all, 550 students were included in this part of the analysis.

Because these students attended schools across the country and took different state assessments, we standardized students' raw test scores to make them comparable across state lines. Each test score was compared to the score needed to be considered to have met the expectations of the grade or course. In many states, this is equivalent to earning a Level 4 out of 5.[26] We then divided the difference between a student's score and this threshold by the standard deviation of all test scores in the state from the same grade and school year.

The correlations between these standardized test scores and the proportion of a student's grade-level ladder completed were high ($r = 0.73$, $p < 0.001$ for all students and $r = 0.75$, $p < 0.001$ for just Algebra I students). And the strength of this relationship held even after controlling for the number of concepts and skills with which students began the year: Increasing the proportion of the ladder completed by 50 percentage points was associated with a 1.5 standard deviation increase in test performance. These results gave us confidence that the record of concept and skill mastery tracked in Teach to One Roadmaps was a strong predictor of state-tested performance.

Identifying Key Predecessors

For each concept and skill in the algebra ladder, we examined every attempt made by Algebra I students to estimate how mastering each predecessor typically changed the student's probability of success, after accounting for where students began the year, how many total predecessors they had already mastered, and how many attempts at that same concept and skill they had already made. In other words, all else equal, how much does a student's chance of mastering algebra concept and skill X change if they had already mastered predecessor concept and skill Y?

To accomplish this, we used separate linear probability models for each skill-predecessor combination[27] on the algebra ladder, controlling for the total number of predecessors mastered, the number of beginning-of-year skills, the number of prior attempts at that skill, and for each predecessor a binary indicator of whether that predecessor was mastered. The three predecessors linked to the largest estimate of the latter became the key predecessors for that skill.

By design, key predecessors strongly predicted the probability of success on any attempt. After controlling for the number of skills a student began the year with, the chances of success on an attempt at a new skill were 36 percentage points higher if the student had mastered all the key predecessors.[28] The strength of this relationship varied by skill, however, with this estimated difference in probability as high as 70 percentage points for some skills and as low as 5 percentage points for others.

Our approach to identifying key predecessors was rooted in testing the concept that a small proportion of predecessor knowledge could have an outsize role in helping students master new concepts and skills. Consequently, we view our identified key predecessors as a starting point for future research on which concepts and skills matter most for learning specific algebra skills and concepts, rather than the definitive list of such. Appendix 1 provides a sample of the key predecessors we identified.

Simulating a School Year

Though Teach to One Roadmaps is intended to supplement core instruction, we used what our analysis revealed about the relationships between predecessor knowledge and the probability of successfully mastering new concepts and skills to simulate the effectiveness of different general instructional approaches over the course of an entire school year. These simulations were not designed to reflect students' progress through Roadmaps but rather a "traditional" classroom where teachers are simultaneously responsible for the learning of many students and for making choices about how to support students who start at different places.

Our simulations were based on actual students' individualized records of success and failure with different types of skills. Using all algebra students' attempts to master algebra concepts and skills, we modeled the relationship between several factors and the probability of success. Specifically, we used a linear probability model predicting attempt success controlling for the proportion of key predecessors mastered, the number of concepts and skills mastered at the beginning of the year, either through a diagnostic assessment and/or previous years in Roadmaps, and their interaction. The results from this model allowed us to estimate the probability that any student would succeed on an attempt given their starting point and the number of key predecessors learned to date.

We then focused on all algebra students who began the year behind—those who began with 75 or fewer skills—and, for each student, identified which of the 147 algebra skills they had mastered at the beginning of the year and which they had not. Then we simulated what would happen if each student made 50 attempts during a school year, using each of the three different strategies described in this report:

1. For the **start-at-the-bottom** approach, we took the mean grade level of all beginning of year concepts and skills and rounded to the nearest integer grade to get the student's initial grade level. Across the 50 attempts, students began with the first concept and skill in their assigned grade-level and moved to the next one regardless of simulated attempt outcome. Concepts and skills in all grade levels were ordered so that students never attempted a concept and skill in a grade without first attempting its same-grade predecessors.
2. For the **grade-level-only** approach, all students began at the first concept and skill of ninth-grade Algebra I and moved to the next concept and skill after each of the 50 attempts, regardless of outcome. Concepts and skills were ordered so that students never attempted a concept and skill in a grade without first attempting its same-grade predecessors.
3. For an **individualized** approach, each concept and skill was sorted by the proportion of key predecessors already completed and then arranged by descending grade level. Students attempted the concept and skill at the top of this list; when there were ties, we randomly selected a concept and skill for a student to attempt. Students could attempt the same concept and skill on the next iteration if they did not pass it on a simulated attempt.

Though our simulations were rooted in actual students' starting points and the data-based relationships between key predecessors and the chance of success, they are nonetheless simplifications of approaches one might take in a Tier 2 setting. This simplification was necessary to provide a clear structure for determining which concept and skill a student should focus on in each simulated attempt. Consequently, these simulations should be viewed as ways to illustrate the importance of attempting new concepts and skills with a solid foundation in key predecessors rather than a precise prescription of a specific Tier 2 strategy.

Though we find our simulation helpful to illustrate the potential yearlong effects of different Tier 2 approaches, we encourage future researchers to ask similar questions with data that has more natural variation in the algebra path through skills and concepts students take. Rather than simulating outcomes based on data-based associations, we encourage others to leverage supplemental curricular data to track the actual outcomes obtained when students approached Tier 2 instruction from these or other perspectives. Given the prevalence of supplemental math programs in schools, there is much that can be learned and shared about how different paths through the algebra ladder tend to lead to different results.

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Endnotes

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7 Students using Roadmaps for algebra tended to be in schools serving higher-than-average proportions of students experiencing poverty. The typical percentage of students receiving free and reduced-price lunch was approximately 82 percent. Schools were highly likely to be eligible for Title I funding (83 percent), had less than the typical proportions of white students (11 percent), and the typical proportion of Black and Hispanic students were 23 percent and 50 percent respectively. We did not have individual student data to look at the makeup of users directly but given that their schools departed meaningfully from national averages, the sample of students in our data should not be viewed as nationally representative.

8 To test this, we modeled students' standardized test scores against the proportion of total concepts and skills mastered prior to taking the state assessment and found that a 50 percentage point increase in the proportion of concepts and skills mastered was associated with a 1.29 standard deviation increase in test performance ($p < 0.001$). For context, the typical difference between grade levels in standardized test scores on NAEP is 0.33 standard deviations (Fahle et al., 2024). To this model we also added controls for the grade-level of the assessment and the proportion of concepts and skills that students had mastered at the beginning of the year and found qualitatively similar associations. See the Methodology section for more details on how we standardized test scores and our modeling approaches.

9 The average test score units for students who knew less than one-third of the eligible skills prior to taking their state assessment was 1.26 standard deviations below the score needed to meet expectations and 1 percent of these students met expectations. The average test score units for students who knew four-fifth or more of the eligible skills prior to taking their state assessment was 0.56 standard deviations above the score needed to meet expectations and 86 percent of these students met expectations.

10 Of the approximately 2,000 students who participated in an Algebra I class, approximately 46 percent began the school year having already mastered no more than 32 algebra skills; there are 95 algebra skills introduced to students prior to Algebra I

11 See the Methodology section for more details on how we used the data to identify the predecessors that were most associated with an attempt's chances of success.

12 This held true regardless of school demographics, including schools serving mostly students experiencing poverty, in schools where nearly all students were Black, and in schools where nearly all students were Hispanic. In all cases, mastering all key predecessors was associated with significantly higher chances of succeeding on a new skill and concept.

13 Because not all grade-level skills and concepts had exactly three key predecessors, we first calculated the proportion of key predecessors to each grade-level skill that were introduced in Grade 7 or higher, then calculated an average across skills weighted by the number of key predecessors.

14 Because the proportion of key predecessors mastered is positively correlated with the proportion of all predecessors mastered, we also used a linear probability model to estimate students' chances of success on attempts after controlling for the number of beginning-of-year concepts and skills mastered, the total proportion of predecessors mastered for the given concept and skill attempted, and the proportion of key predecessors mastered for that concept and skill. From this model, going from 0 percent to 100 percent mastery over key predecessors was associated with a 36-percentage-point increase in the chances of success, even after these controls. This association was statistically significant ($p<0.001$).

15 To reinforce this, we pooled all attempts at a skill in the algebra ladder and used a linear probability model to estimate the difference in the chances of success if a student knew all key predecessors vs. none, after controlling for the total number of skills already mastered at the beginning of the year (N = 128,473). Knowing all key predecessors increased the probability of success by 41 percentage points ($p < 0.001$).

16 For all simulations to follow, we isolated students who began Algebra I demonstrating mastery (via a diagnostic assessment and/or from previous school years) of 75 or fewer algebra-related concepts and skills.

17 See Methodology section for more details on our simulation approach, including how we defined the different scenarios described below.

18 For example, one estimate says that 64 percent of districts use between one and three K-5 math supplemental products, which tend to have diagnostic features. The Center for Education Market Dynamics. (2024, October). Beyond the Core: National Trends in District Selection of K-5 Supplemental Math Products. https://www.cemd.org/beyond-the-core-national-trends-in-district-selection-of-k-5-supplemental-math-products/

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21 Only 47 grade-level algebra skills had at least one key predecessor that was also rated highly by New Classrooms separate assessment.

22 Only 47 grade-level algebra skills had at least one key predecessor that was also rated highly by New Classrooms separate assessment.

23 Students not in Algebra I classrooms could also attempt algebra-related skills because some of these skills are core parts of mathematics from earlier grades and/or some students attempt content above their grade level. However, in order to have our results better reflect the full Algebra I experience, we only looked at attempts made by students enrolled in an official Algebra I class. Most (66 percent) of these students were in ninth grade and 22 percent were in 10th grade.

24 These attempts include concepts and skills mastered during diagnostic assessments (74,538), attempted during the school year (50,385), and concepts and skills inferred to have been mastered because students successfully demonstrated a closely related higher-level concept or skill (3,656).

25 Teachers have the option of having students complete diagnostic assessments later in the school year too. To ensure we were capturing beginning-of-year skill mastery, we isolated diagnostic assessments given within the first 30 days of beginning Roadmaps for the year.

26 In some states, "meeting expectations" is a higher performance level than simply passing the test.

27 We only included combinations where there were at least 25 attempts made when the predecessor was already mastered and at least 25 attempts made when the predecessor was not mastered. When identifying key predecessors, we also included results from the diagnostic assessments because they provided a record of concept and skill mastery and a simultaneous snapshot of which predecessor skills students had already mastered.

28 We pooled all concepts and skills and used a linear probability model to estimate the effect of having already mastered all three key predecessors after controlling for the total number of beginning-of-year concepts and skills. Estimate was significantly different from zero ($p < 0.001$).

About TNTP

As a leading education nonprofit since 1997, TNTP combines insight, courage, and action to conceive scalable solutions that address complex challenges from the classroom to the systems level. Today, we work side by side with educators, system leaders, and communities across 41 states and more than 4,000 school systems nationwide to reach ambitious goals for student success. Our vision pushes beyond school walls, catalyzing cross-sector collaboration to create pathways for young people to achieve academic, economic, and social mobility.